Information and Computation **148**, 241–244 (1999)
Article ID inco.1998.2758, available online at http://www.idealibrary.com on IDEAL®

CORRIGENDUM

Volume **122**, Number 1 (1995), in the article "On Some Decision Problems in Programming" by Dieter Spreen, pages 120–139: The paper is based on an approach to effectively given topological T_0-spaces, where, instead of the usual set inclusion between basic open sets, a strong inclusion relation is used. But the strong inclusion relation presented in Example 2.7 is not a relation between basic open sets, yet only between their indices. Take for example the set of all natural numbers with the discrete metric and let the indexing β be the identical mapping. Then one has that $B_{\langle i, 1\rangle} \prec B_{\langle i, 0\rangle}$. But since $B_{\langle i, 1\rangle} = \{i\} = B_{\langle i, 0\rangle}$, one should have also that $B_{\langle i, 0\rangle} \prec B_{\langle i, 1\rangle}$, which is not the case. Therefore one has to base the approach on a relation between indices of basic open sets.

Definition 1. Let $\mathscr{T} = (T, \tau)$ be a countable topological T_0-space with a countable basis $\mathscr{B}$ and let B be a numbering of $\mathscr{B}$. Moreover, let $\prec_B$ be a transitive binary relation on ω. We say that:

1. $\prec_B$ is a *strong inclusion*, if for all $m, n \in \mathrm{dom}(B)$, from $m \prec_B n$ it follows that $B_m \subseteq B_n$.

2. $\mathscr{B}$ is a *strong basis*, if $\prec_B$ is a strong inclusion and for all $z \in T$ and $m, n \in \mathrm{dom}(B)$ with $z \in B_m \cap B_n$ there is a number $a \in \mathrm{dom}(B)$ such that $z \in B_a$, $a \prec_B m$, and $a \prec_B n$.

All further definitions, where the relation $\prec$ is used, have to be changed appropriately. Moreover, instead of complete enumerability one has to require that $\prec_B$ is r.e., similar for complete recursivity. The operation hl is applied only to basic open sets. It has to be redefined as

$$\mathrm{hl}(B_n) = \bigcap \{B_m \mid n \prec_B m\}.$$

Moreover, just as the notion of a topological basis had to be adjusted to the use of a strong inclusion relation, the same holds for that of a filter base.

Definition 2. Let $\mathscr{H}$ be a filter. A nonempty subset $\mathscr{F}$ of $\mathscr{H}$ is called *strong base* of $\mathscr{H}$ if the following two conditions hold:

1. For all $m, n \in \mathrm{dom}(B)$ with $B_m, B_n \in \mathscr{F}$ there is some index $a \in \mathrm{dom}(B)$ such that $B_a \in \mathscr{F}$, $a \prec_B m$, and $a \prec_B n$.

2. For all $m \in \mathrm{dom}(B)$ with $B_m \in \mathscr{H}$ there is some index $a \in \mathrm{dom}(B)$ such that $B_a \in \mathscr{F}$ and $a \prec_B m$.

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If $(B_{f(a)})_{a\in\omega}$ is a normed enumeration of basic open sets, we say that it *converges* to some point in case it enumerates a strong base of the neighbourhood filter of this point. A T_0-space with a countable basis is *constructively complete* if each normed recursive enumeration of nonempty basic open sets converges.

Observe that now in the definition of constructive A- and f-spaces one has to require that the neighbourhood filter of each point has an enumerable strong base of basic open sets. As a consequence of this the proofs of Lemmas 2.13 and 2.17 have to be modified.

For Lemma 2.13, if $\{B_a \mid a\in W_i\}$ is a strong base of the neighbourhood filter of some point y set $x_i = y$; otherwise let x_i be undefined. With this change the proof proceeds as in the paper, but note that in order to verify that the numbering x is computable one does not have to use that the strong inclusion relation is closed under set inclusion, which, in fact, is not true. By the properties of a strong filter base and the construction of the function f we have that

$$\begin{aligned} x_i\in B_n &\Rightarrow (\exists c\in W_i)\; c \prec_B n\\ &\Rightarrow (\exists a\in \mathrm{range}(\varphi_{f(i)}))(\exists c\in W_i)\; a\prec_B c\prec_B n\\ &\Rightarrow (\exists a\in \mathrm{range}(\varphi_{f(i)}))\; a\prec_B n. \end{aligned}$$

If, on the other hand, $a\prec_B n$, for some $a\in\mathrm{range}(\varphi_{f(i)})$, then $a\in W_i$ and, hence, $x_i\in B_n$ since $\{B_a \mid a\in W_i\}$ is a strong base of the neighbourhood filter of x_i. Thus,

$$x_i\in B_n \Leftrightarrow (\exists a\in\mathrm{range}(\varphi_{f(i)}))\; a\prec_B n.$$

In the proof of Lemma 2.17 it was claimed that, if $f(m)$ is an index of a normed recursive sequence of elements of the dense base M_0, converging to some point $z\in M$, then $(B(\varphi_m(a)))_{a\in\omega}$ is a normed recursive enumeration of basic open sets which converges to z. Here, we have to show, in addition, that for each basic open set B_c with $z\in B_c$ there is some number a such that $\varphi_m(a)\prec_B c$.

Let $c\in\omega$ such that $z\in B_c$. Then there are $a, a'\in\omega$ with $a\leqslant a'$ such that $\delta(\beta(\varphi_{f(m)}(a)), \beta(\varphi_{f(m)}(a'))) < 2^{-a}$, which implies that $\delta(\beta(\varphi_{f(m)}(a)), z)\leqslant 2^{-a}$. Moreover, we have that $\delta(\beta(\pi_1(\varphi_m(a))), \beta(\pi_1(\varphi_m(a+1)))) + 2^{-\pi_2(\varphi_m(a+1))} < 2^{-\pi_2(\varphi_m(a))}$. It follows that $\pi_2\circ\varphi_m$ is strictly increasing. Now, choose $a\in\omega$ such that $2^{-a} < 2^{-\pi_2(c)-1} - 2^{-1}\cdot\delta(z, \beta(\pi_1(c)))$ and $2^{-\pi_2(\varphi_m(a))} < 2^{-\pi_2(c)-1} - 2^{-1}\cdot\delta(z, \beta(\pi_1(c)))$. Observe here that $\delta(z, \beta(\pi_1(c))) < 2^{-\pi_2(c)}$, since $z\in B_c$. Then we have that

$$\begin{aligned} &\delta(\beta_{\pi_1(\varphi_m(a))}, \beta_{\pi_1(c)}) + 2^{-\pi_2(\varphi_m(a))}\\ &\quad\leqslant \delta(\beta_{\pi_1(\varphi_m(a))}, z) + \delta(z, \beta_{\pi_1(c)}) + 2^{-\pi_2(\varphi_m(a))}\\ &\quad\leqslant 2^{-a} + \delta(z, \beta_{\pi_1(c)}) + 2^{-\pi_2(\varphi_m(a))}\\ &\quad< 2^{-1}\cdot(2^{-\pi_2(c)} - \delta(z, \beta_{\pi_1(c)})) + \delta(z, \beta_{\pi_1(c)}) + 2^{-1}\cdot(2^{-\pi_2(c)} - \delta(z, \beta_{\pi_1(c)}))\\ &\quad= 2^{-\pi_2(c)}. \end{aligned}$$

Moreover, it was claimed that, if m is an index of a normed recursive sequence of elements of M_0 which converges to some point z, then $(B(\varphi_{k(m)}(a)))_{a\in\omega}$ is a normed recursive enumeration of basic open sets converging to z. Here too, it remains to show that for every basic open set B_c with $z \in B_c$ there is some number a such that $\varphi_{k(m)}(a) \prec_B c$.

By the definition of the function k we have that

$$\begin{aligned}\delta(\beta_{\pi_1(\varphi_{k(m)}(a))}, \beta_{\pi_1(c)}) + 2^{-\pi_2(\varphi_{k(m)}(a))} &= \delta(\beta_{\varphi_m(a+1)}, \beta_{\pi_1(c)}) + 2^{-a} \\ &\leqslant \delta(\beta_{\varphi_m(a+1)}, z) + \delta(z, \beta_{\pi_1(c)}) + 2^{-a}.\end{aligned}$$

Since $(\beta(\varphi_m(a)))_{a \in \omega}$ converges to z, there is some number $\bar{a}$ such that $\delta(\beta(\varphi_m(e+1)), z) < 2^{-1} \cdot (2^{-\pi_2(c)} - \delta(z, \beta(\pi_1(c)))$ for all $e \geqslant \bar{a}$. Furthermore, there is some number $\tilde{a}$ with $2^{-\tilde{a}} < 2^{-1} \cdot (2^{-\pi_2(c)} - \delta(z, \beta(\pi_1(c)))$. Choosing a to be the maximum of $\bar{a}$ and $\tilde{a}$ we obtain that

$$\delta(\beta_{\varphi_m(a+1)}, z) + \delta(z, \beta_{\pi_1(c)}) + 2^{-a} < 2^{-\pi_2(c)}.$$

As has already been said, in contradiction to what is stated in the paper, the relations $\prec$ defined in Examples 2.4 and 2.6 are not closed under set inclusion. If, however, in these examples strong inclusion relations are defined in the same way, then these relations are correct.

Definition 3. A strong inclusion relation $\prec_B$ is called *correct* if for every finite point $y \in T$ there is some index c such that $B_c = \{z \in T \mid y \leqslant_\tau z\}$ and $c \prec_B a$, for all $a \in \mathrm{dom}(B)$ with $y \in B_a$.

In what follows we show that in the results where closure under set inclusion was assumed this condition can be replaced by the supposition that the strong inclusion relation is correct.

Let us start with Lemma 2.25. Here we need, in addition, a further refinement of the property to be effectively pointed.

Definition 4. $\mathscr{T}$ is *strongly pointed* if it is effectively pointed and the function pd satisfies the additional requirement that for $m, n \in \mathrm{dom}(B)$ with $x_{\mathrm{pd}(n)} \in B_m$ one has that $n \prec_B m$.

Note that constructive A- and f-spaces, as well as constructive domains are strongly pointed.

Lemma 2.25. *Let $\mathscr{T}$ be strongly and honestly pointed, let all points $x_{\mathrm{pd}(n)}$ be finite, and let T have a smallest element. Moreover, let $\prec_B$ be correct. Then B is extensional.*

Proof. As in the paper it follows that T is a basic open set, say B_a. Since $\mathscr{T}$ is strongly pointed, it follows that $m \prec_B a$, for all $m \in \omega$. Now, we can follow the argument given in the paper. We only have to show in addition that there is some index b such that $B_b = \{z \in T \mid x_{\mathrm{pd}(a_i)} \leqslant_\tau z\}$ and $a_i \prec_B b \prec_B a_{i-1}$. Since $x_{\mathrm{pd}(a_i)} \in B_{a_i - 1}$, $x_{\mathrm{pd}(a_i)}$ is finite, and $\prec_B$ is correct, there is some index b such that $b \prec_B a_i - 1$ and $B_b = \{z \in T \mid x_{\mathrm{pd}(a_i)} \leqslant_\tau z\}$. As $\mathscr{T}$ is strongly pointed, this implies that also $a_i \prec_B b$.

LEMMA 4.7. *Let y be finite and neither minimal nor maximal. Moreover, let x be computable, and let $\prec_B$ be recursive and correct. Then* $(\Omega(\{y\}), \Omega(\overline{\{y\}})) \leqslant_1 (K \times \bar{K}, \overline{K \times \bar{K}})$.

Proof. The proof proceeds as in the paper. We only have to verify that there is some index c such that $B_c = \{z \in T \mid y \leqslant_\tau\}$ and $y \in B_n$ if and only if $c \prec_B n$, for all $n \in \omega$, which is obvious since y is finite and $\prec_B$ is correct.

Note that Lemma 2.25 and Lemma 4.7, respectively, are used in the proofs of Corollary 6.7 and Theorem 4.9. The assumptions in both statements have to be adjusted. All other statements remain true.

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